Hong Kong International Journal of Research Studies, ISSN: 3078-4018
Volume 2, Issue 2, July December, 2024
Available online at: https://octopuspublication.com/index.php/hkijrs

Historical Exploration of Number Theory and its Modern Applications

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The study of integers, and more especially positive integers, is referred to as number theory in the branch of mathematics known as mathematics. Since quite some time ago, its relevance in mathematics has been well-established and well recognised. For the simple reason that this is a subject that has a great deal of historical relevance. Unlike the majority of other fields, this one allows for the observation of results. From the time of antiquity till the present day, humanity have been able to develop new and exciting insights into the nature of numbers in each and every century. The great majority of the world's most accomplished mathematicians have, throughout the course of their careers, made substantial contributions to the field of number theory. Because of the basic qualities that it has, number theory has captured the interest of the most prominent scientists.

A branch of mathematics known as number theory, which is often referred to as the "Queen of Mathematics," examines the characteristics and relationships of integers. Included in the list of possible predecessors are the first mathematical publications, which focused on basic concepts related to numbers. There is a set of equations known as Diophantine equations that are looking for integer solutions. Number theory has developed and changed over the course of thousands of years, and it has been related to these equations. This introduction aims to outline the historical progression of number theory and the central role played by Diophantine equations, setting the stage for contemporary research.

Many people regard Pythagoras, a Greek mathematician and physicist, to be the "father of number theory" because of the ground-breaking work he did in the domains of geometry and number theory. The Pythagorean theorem, which is concerned with the sides of a right triangle, is considered to be one of the most significant discoveries that Pythagoras and his pupils made in the field of number theory. In the discipline of number theory, Euclid, Fermat, and Diophantus are famous personalities who have made substantial contributions to the study of equations, number systems, and prime numbers. These individuals have also made significant contributions to the field. Even in this day and age, having a solid understanding of number theory in the context of mathematics is essential and useful.

Number theory is a topic of mathematics that encompasses everything that has to do with numbers, including their underlying characteristics, operations, and characteristics of their nature. For hundreds of years, mathematicians have been fascinated by this fascinating subject in order to satisfy their curiosity. In addition to the domains of engineering and general science, it may also be beneficial to the subject of computer science.

A fundamental focus of number theory is the study of numbers and the characteristics that distinguish them from one another. Congruences, prime numbers, diophantine equations, and the capacity to divide are a few instances of this kind of talent. A prime number is a positive integer that can only be divided by itself and by one. Prime numbers are an example of this. It is one of the many forward-thinking ideas in number theory, and it has a prominent position. The numbers two, three, five, seven, eleven, and thirteen are among the prime numbers that were created first.

In number theory, prime numbers are the topic of a significant amount of research because of the many fascinating properties that they possess. A well-known theory known as the "twin prime conjecture" asserts that there is an endless number of prime number pairs that are not identical to one another, such as three and five. In spite of the absence of evidence, mathematicians have made significant progress in their understanding of the distribution of prime numbers and the characteristics they possess.

However, this is yet another essential notion in the field of number theory. Specifically, the focus of this area of study is on the process of dividing numbers by other integers. In this area of research, the study of prime numbers is an essential component. The fact that the integer n can be divided by the integer m may be deduced from the fact that a number k can be found such that n = km. There are a number of domains, including computer science and security, that might potentially benefit from the use of this approach. There are also a great deal of mathematical applications that may be made use of it.

Number theory also includes congruences, which are an important mathematical concept. It is necessary for two numbers to have the same result when divided by a predefined integer in order for them to be termed congruent. The reason why 12 and 2 are comparable to 2 (mod 5) is due to the fact that when divided by 5, each of these numbers

contain a residue of 2. Not only are congruences useful in the process of learning modular mathematics, but they are also very important in the fields of computer science and software security.

Diamond equations are also included in the field of number theory. These equations, which are based on integers, are presented as a tribute to the Greek scholar Diophantus. Despite the fact that Diophantine equations are notoriously difficult, some well-known scientists have spent years seeking to find a solution to them.

Number theory has a wide range of exciting applications, some of which include quantum physics, engineering, and computer science. In spite of the fact that the ideas and concepts that underpin number theory seem to be ambiguous and complicated, the area has a long and illustrious history, and it has been an indispensable contributor to the advancement of mathematics and science over the course of many centuries. Expanding one's knowledge in the fascinating topic of number theory would be beneficial for everyone, from scholars and scientists to just interested members of the general public.

1. Ancient Beginnings

The origins of number theory are found in ancient Egypt and Babylon, where arithmetic techniques were first systematically developed. The Rhind Mathematical Papyrus (circa 1650 BCE) shows early work on fractions and linear equations. In ancient Greece, the Pythagoreans revered numbers as fundamental to understanding the cosmos. Euclid's Elements (circa 300 BCE) introduced the first systematic study of divisors and prime numbers, along with the Euclidean algorithm for computing the greatest common divisor (GCD).

Diophantus of Alexandria, in his seminal work Arithmetica (circa 250 CE), introduced the concept of Diophantine equations. These equations, often nonlinear, sought solutions in the realm of integers, such as the classic equation ax+by=cax + by = cax+by=c.

LITERATURE REVIEW

Pythagorean triangles attracted many mathematicians. **Gopalan & Devibala,** (2005) discussed a special type of Pythagorean triangles in which the area of the triangle, with sides (x,y,z) such that $x^2 + y^2 = z^2$, is equal to the perimeter of the triangle numerically i.e. $\frac{1}{2}xy = x + y + z$. They have shown that there exist one primitive integral solution and one non-primitive integral solution of such triangles.

In this chapter, the Pythagorean triangles have been discussed for other relations concerning their area and perimeter. Besides Pythagorean triangles, the Diophantine equations relating area and perimeter of rectangle, circle and cardioid and surface area and volumes of rectangular parallelepiped, sphere, cylinder, and cone have been discussed. The integral solutions have been obtained under different conditions.

Diophantine equations are polynomial equations that seek integer solutions. Many of these equations arise naturally from geometrical contexts, especially when dealing with properties of standard geometrical figures like triangles, circles, ellipses, and squares. Below is an exploration of how Diophantine equations are connected to these figures.

1. Right Triangles and Pythagorean Triples

A right triangle satisfies the Pythagorean theorem:

a2+b2=c2a^2 + b^2 = c^2a2+b2=c2

where aaa, bbb, and ccc are the lengths of the two legs and the hypotenuse, respectively. Solutions to this equation in positive integers are called Pythagorean triples.

- Primitive Pythagorean Triples: Generated by the formulas: a=m2−n2,b=2mn,c=m2+n2a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2a=m2−n2,b=2mn,c=m2+n2 where mmm and nnn are coprime integers with m>n>0m > n > 0m>n>0 and m−nm - nm−n is odd.

Example:

(3,4,5)and(5,12,13)(3, 4, 5) \quad \text{and} \quad (5, 12, 13)(3,4,5)and(5,12,13)

2. Squares and Sum of Two Squares

A classic problem in number theory is determining which integers can be expressed as the sum of two squares:

x2+y2=nx^2 + y^2 = nx2+y2=n

Fermat's Two-Square Theorem:

- A number nnn can be expressed as a sum of two squares if and only if every prime ppp of the form 4k+34k + 34k+3 in the prime factorization of nnn appears with an even exponent.

Geometrical Context:

- This equation describes all points (x,y)(x, y)(x,y) on a circle of radius n\sqrt{n}n centered at the origin.

3. Ellipses and Elliptic Curves
An ellipse in its standard form is represented by:
x2a2+y2b2=1\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1a2x2+b2y2=1
However, elliptic curves relevant to Diophantine analysis take the form:
y2=x3+ax+by^2 = x^3 + ax + by2=x3+ax+b
These curves have rich connections to number theory, particularly in solving Diophantine equations.
Applications:

- Fermat's Last Theorem: Proved by linking solutions of xn+yn=znx^n + y^n = z^nxn+yn=zn to properties of elliptic curves.
- Mordell-Weil Theorem: Demonstrates that the set of rational points on an elliptic curve forms a finitely generated group.

4. Pell's Equation and Rectangles
The Pell equation:
x2−Ny2=1x^2 - Ny^2 = 1x2−Ny2=1
can be interpreted geometrically. If we consider a rectangle with dimensions xxx and Ny\sqrt{N}yNy, the equation ensures a near-square ratio for rational approximations of N\sqrt{N}N.

- Solution Method: Continued fractions provide a systematic way to generate integer solutions (x,y)(x, y)(x,y).

5. Diophantine Problems in Triangles
The Heronian triangle problem seeks integer-sided triangles with integer area.

- Heron's Formula for the area AAA of a triangle with sides aaa, bbb, and ccc: A=s(s−a)(s−b)(s−c),s=a+b+c2A = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a + b + c}{2}A=s(s−a)(s−b)(s−c),s=2a+b+c For AAA to be an integer, s(s−a)(s−b)(s−c)s(s-a)(s-b)(s-c)s(s−a)(s−b)(s−c) must be a perfect square.

Example: A triangle with sides 5,12,135, 12, 135,12,13 has an integer area of 30.

6. Circles and Integer Points
For a circle centered at the origin, the equation is:
x2+y2=r2x^2 + y^2 = r^2x2+y2=r2
Finding integer solutions corresponds to determining lattice points lying on the circle.
Geometrical Interpretation:

- Solutions describe points where a circle intersects the integer lattice. This connects with sums of two squares, where r2r^2r2 must be expressible as x2+y2x^2 + y^2x2+y2.

7. Cuboids and Perfect Cubes
In three dimensions, problems involve cuboids or integer solutions for volumes and diagonal lengths:
x2+y2+z2=w2x^2 + y^2 + z^2 = w^2x2+y2+z2=w2
This equation represents the distance from the origin to a point in 3D space.

- Perfect Cuboid Problem: Does a cuboid exist with integer side lengths, face diagonals, and space diagonals? This remains an open problem in number theory.

Diophantine equations have deep connections with geometry, providing insights into classical problems involving triangles, circles, and ellipses. These equations not only solve integer-coordinate problems but also reveal intricate links between algebraic and geometric properties.

3. NUMBER OF LATTICE POINTS WHITHIN AND ON SOME STANDARD TWO AND THREE DIMENSIONAL GEOMETRICAL FIGURES

Number Theory is classified into four categories: Elementary Number Theory (or Classical Number Theory), Analytic Number Theory, Algebraic Number Theory and Geometric Number Theory. Geometric Number Theory is an important class of Number Theory. In Geometric Number Theory, we deal with the problem of Number Theory by geometric methods. In a plane orthogonal coordinate system, a point (x,y) is called an integral part (or lattice point) if its coordinates x and y are positive integers. In this class of Number Theory we have to find lattice points in the given closed figure. A very famous and difficult unsolved problem in Number Theory is Gauss's integral point problem stated as: How many integral points are there inside the circle with center at the origin and radius r? If A(r) is the number of integral points within and on the circle then Gauss's problem is to seek the relationship between A(r) and r. Since the area of the circle with radius r is , it is conjectured that . Some results regarding this problem are as follows:

Result given by **Sierpinski** is $A(r) = \pi r^2 + o\left(r^{2/3}\log r\right)$.

Result given by **Lu-Keng Hua** is $A(r) = \pi r^2 + o\left(r^{13/20}(\log r)^{9/8}\right)$.

Result given by **Jing-Run Chen** is $A(r) = \pi r^2 + o\left(r^{\frac{3}{4}+8}\right)$. This is the best result known till now.

Minkowski showed that there must exist a non-zero integral point inside symmetric convex whose volume is greater than . In this chapter, an attempt will be made to obtain the number of lattice points within and on some two dimensional and three dimensional geometrical closed figures such as circle, ellipse, cardioid, square, rectangle, sphere, cuboid and rectangular parallelopiped etc. by considering their equations as Diophantine equations.

Lattice points are points in the Euclidean space Rn\mathbb{R}^nRn whose coordinates are integers. The problem of counting lattice points within and on the boundary of various two- and three-dimensional geometrical figures has been a classical subject in number theory and geometry. Below, we explore this problem for standard shapes like circles, squares, spheres, and cubes.
1. Lattice Points in Two Dimensions
1.1 Circle
The equation of a circle with radius rrr centered at the origin is:
x2+y2=r2x^2 + y^2 = r^2x2+y2=r2
Lattice Points on the Circle
To find integer solutions (x,y)(x, y)(x,y) such that x2+y2=r2x^2 + y^2 = r^2x2+y2=r2, we need r2r^2r2 to be expressible as the sum of two squares:
r2=x2+y2r^2 = x^2 + y^2r2=x2+y2

- Key Insight: If r2r^2r2 is a sum of two squares, the number of lattice points on the circle is related to the divisor structure of r2r^2r2.

For each valid pair (x,y)(x, y)(x,y), there are 4 symmetric points:
(x,y),(−x,y),(x,−y),(−x,−y)(x, y), \quad (-x, y), \quad (x, -y), \quad (-x, -y)(x,y),(−x,y),(x,−y),(−x,−y)
Lattice Points Inside the Circle
To count lattice points (x,y)(x, y)(x,y) such that:
x2+y2≤r2x^2 + y^2 \leq r^2x2+y2≤r2
One can estimate this count using Gauss's circle problem, where the number of lattice points N(r)N(r)N(r) is approximated by:
N(r)∼πr2N(r) \sim \pi r^2N(r)∼πr2
However, there is an error term:
N(r)=πr2+E(r)N(r) = \pi r^2 + E(r)N(r)=πr2+E(r)
where E(r)=O(r)E(r) = O(r)E(r)=O(r). This error term accounts for the deviation due to the discrete nature of lattice points.
1.2 Square
For a square centered at the origin with side length 2a2a2a, the boundary is defined by:
|x|≤a,|y|≤a|x| \leq a, \quad |y| \leq a|x|≤a,|y|≤a
Lattice Points on the Square

- The lattice points on the boundary can be easily counted: Nboundary=8a+4N_{\text{boundary}} = 8a + 4Nboundary=8a+4 (4 corners, 4(a−1)4(a - 1)4(a−1) points per side).

Lattice Points Inside the Square
The total number of lattice points inside (including the boundary) is:
Ntotal=(2a+1)2N_{\text{total}} = (2a + 1)^2Ntotal=(2a+1)2
Subtracting the boundary points gives:
Ninside=(2a+1)2−(8a+4)N_{\text{inside}} = (2a + 1)^2 - (8a + 4)Ninside=(2a+1)2−(8a+4)

2. Lattice Points in Three Dimensions
2.1 Sphere
The equation of a sphere with radius rrr centered at the origin is:
x2+y2+z2=r2x^2 + y^2 + z^2 = r^2x2+y2+z2=r2
Lattice Points on the Sphere
To find integer solutions (x,y,z)(x, y, z)(x,y,z) such that:
x2+y2+z2=r2x^2 + y^2 + z^2 = r^2x2+y2+z2=r2
requires r2r^2r2 to be expressible as a sum of three squares. A necessary condition is that r2r^2r2 should not be of the form 4k(8m+7)4^k(8m + 7)4k(8m+7).
The number of solutions is more complex than in two dimensions. It depends on the divisor count of r2r^2r2 in relation to sums of squares.
Lattice Points Inside the Sphere
The problem of counting lattice points inside a sphere is analogous to the three-dimensional version of Gauss's circle problem:
N(r)∼43πr3N(r) \sim \frac{4}{3} \pi r^3N(r)∼34πr3
with an error term:

E(r)=O(r2)E(r) = O(r^2)E(r)=O(r2)
2.2 Cube
For a cube centered at the origin with side length 2a2a2a, the boundaries are defined by:
|x|≤a,|y|≤a,|z|≤a|x| \leq a, \quad |y| \leq a, \quad |z| \leq a|x|≤a,|y|≤a,|z|≤a
Lattice Points on the Cube
The number of lattice points on the boundary involves contributions from:

- Corners: 888 points
- Edges: 12×(2a−1)12 \times (2a - 1)12×(2a−1)
- Faces: 6×(2a−1)26 \times (2a - 1)^26×(2a−1)2

Lattice Points Inside the Cube
The total number of lattice points (including the boundary) is:
Ntotal=(2a+1)3N_{\text{total}} = (2a + 1)^3Ntotal=(2a+1)3
To find the number inside, subtract the boundary points:
Ninside=Ntotal−NboundaryN_{\text{inside}} = N_{\text{total}} - N_{\text{boundary}}Ninside=Ntotal−Nboundary
3. Asymptotic Formulas and Error Terms
For general convex shapes, the number of lattice points is often approximated by the volume of the shape. However, discrete lattice structures introduce an error term. For a convex region R\mathcal{R}R, let:
N(R)=#{(x1,x2,…,xn)∈Zn:(x1,x2,…,xn)∈R}N(\mathcal{R}) = \#\{(x_1, x_2, \dots, x_n) \in \mathbb{Z}^n : (x_1, x_2, \dots, x_n) \in \mathcal{R}\}N(R)=#{(x1,x2,…,xn)∈Zn:(x1,x2,…,xn)∈R}
The count can be approximated by:
N(R)∼Volume(R)+O(Surface Area)N(\mathcal{R}) \sim \text{Volume}(\mathcal{R}) + O(\text{Surface Area})N(R)∼Volume(R)+O(Surface Area)

4. Applications

- Number Theory: Counting lattice points is related to divisor sums and sums of squares.
- Cryptography: Lattice-based cryptography relies on the structure of high-dimensional lattice points.
- Physics: Models like the Ising model and crystal lattice structures involve lattice point enumeration.

Counting lattice points within and on standard geometrical figures highlights the intersection of geometry, number theory, and asymptotic analysis. These problems reveal deep insights into the distribution of integer solutions and continue to inspire research in pure and applied mathematics.

4. SOME MISCELLENIOUS NON-LINEAR DIOPHANTINE EQUATIONS

There are so many non-linear Diophantine equations. Among which few has been discussed in the previous chapters. The Diophantine equations may or may not have integral solutions. In this chapter, an attempt will be made to discuss some more non-linear Diophantine equations.

A. The Diophantine Equation $x^2 + 7^{2k+1} = y^n$:

Cohn (1993) solved the Diophantine equation $x^2 + 5 = y^n$. **Arif, et al.** (1998) solved the equation $x^2 + 3^{2k+1} = y^n$ completely. **Fadwa, S., Muriefah, A. and Arif, A.** (1999) solved the equation $x^2 + 5^{2k+1} = y^n$. They showed that the equation has no integralsolution in x, y for $n \geq 3, k \geq 0$. **Ramrnujan** (1913) discussed the Diophantine equation $x^2 + 7 = 2^n$ and conjectured that this equation has only five solutions. In the thirdchapter, the generalized **Ramanujan's** equation $x^2 + p = 2^n$, p is a prime number, has been discussed. In this section, the Diophantine equation by Ramanujan has been generalized to $x^2 + 7^{2k+1} = y^n$. For k=0 and y=2 the equation reduces to the **Ramanujan's** Diophantine equation.

If we put x=13, k=1, y=2 and n=9 then the equation $x^2 + 7^{2k+1} = y^n$ is satisfied. Thus (x,k,y,n)= (13,1,2,9) is a solution of the given Diophantine equation.

Theorem 1:

The possible solution (x,y) of the Diophantine equation $x^2 + 7^{2k+1} = y^n$ can not be simultaneously even or odd for any integral value of k and n.

Proof. If x is even then obviously left hand side of the given Diophantine equation is odd and therefore right hand side can not be even. This implies that y cannot be even. If x is odd then left hand side is obviously even and therefore right hand side can not be odd. This implies that y can not be odd. Thus the possible solution of the given Diophantine equation can not be simultaneously even or odd for any integral values of k and n.

Theorem 2:

If x is even and y is odd in the Diophantine equation $x^2+7^{2k+1}=y^n$ then $x^2\equiv 0(\mathrm{mod}8)$ or $x^2\equiv 4(\mathrm{mod}8)$ according as x=4m, m is an integer or x=2m, m is an odd integer and $y^n\equiv y(\mathrm{mod}8)$ or $y^n\equiv 1(\mathrm{mod}8)$ according as n is odd or even.

Proof. If x is an even number and is of the form x=4m, m is any integer then x^2 is of the form $16m^*$, m^* is a positive integer, which is divisible by 8. Therefore $x^2\equiv 0(\mathrm{mod}8)$. If x is even and is of the form x=2m, m is an odd integer then x^2 is of the form $4m^*$, m^* is an odd integer, then obviously the remainder is 4 when x^2 is divided by 8. Therefore $x^2\equiv 4(\mathrm{mod}8)$. Further if we take y=3 and n=3 then $y^n=3^3=27\equiv 3(\mathrm{mod}8)$. Similarly if we take y=5 and n=3 then $y^n=5^3=125\equiv 5(\mathrm{mod}8)$. Thus it can be shown that $y^n\equiv y(\mathrm{mod}8)$ when y is odd and n is odd. If we take y=3 and n=4 then $y^n=3^4=81\equiv 1(\mathrm{mod}8)$. If we take y=5 and n=4 then $y^n=5^4=625\equiv 1(\mathrm{mod}8)$. Thus it can be shown that $y^n\equiv 1(\mathrm{mod}8)$ when y is odd and n is even.

Theorem 3:

If x is odd and y is even in the Diophantine equation $x^2+7^{2k+1}=y^n$ then $x\equiv 1(\mathrm{mod}8)$, $x^2+7^{2k+1}\equiv 0(\mathrm{mod}8)$ and $y\equiv 0(\mathrm{mod}8)$ for $n\geq 3$.

Proof. If we take x=3 and k=1 then $x^2=3^2=9=1(\mathrm{mod}8)$ and $x^2+7^{2k+1}=3^2+7^3$ =9+343=352==0(mod8). Similarly if we take x=5 and k=1 then $x^2=5^2=25=1(\mathrm{mod}8)$ and $x^2+7^{2k+1}=5^2+7^3$ =25 +343=368=0(mod8).

In this way it can be shown that if x is an odd integer then $x\equiv 1(\mathrm{mod}8)$, $x^2+7^{2k+1}\equiv 0(\mathrm{mod}8)$. If y is even, say, 2k and $n\geq 3$ then $y^n=8k^*\equiv 0(\mathrm{mod}8)$.

The present work is devoted to the discussion of some Diophantine equations. Diophantine equation is an important part of Number Theory which is one of the oldest branches of Mathematics. Most of the great masters of Mathematical Sciences, at some point in their careers, have contributed to Number Theory. In this Thesis an attempt has been made to solve some Diophantine equations. This Thesis contains seven chapters.

In the **first chapter**, the concepts and historical background of Number Theory and Diophantine equations have been given. Some survey work related to the development of Diophantine equations in last few decades has also been given in this chapter. A brief summary of the work done in this Thesis has also been given in the end of this chapter.

In the **fourth chapter**, the Diophantine equations related to some standard geometrical figures have been discussed. In this chapter, the Pythagorean triangles have been discussed for different relations concerning their area and perimeter. Besides Pythagorean triangles, the Diophantine equations relating area and perimeter of equilateral triangle, rectangle, circle and cardioid and surface area and volumes of rectangular parallelepiped, sphere, cylinder, cone and regular polyhedrons have been discussed. The integral solutions have been obtained under different conditions. Some results obtained in this chapter are as follows:

The Diophantine equations related with the area and perimeter of the Pythagorean triangles have been discussed in five cases, i.e. A=2P, 2A=P, A=3P, A=Pp and 2A=Pp where A is the area of the Pythagorean triangle, P is its perimeter and p is a prime number. The Diophantine equations related with the area and perimeter of the rectangle have been discussed in five cases, namely, A=P, 2A=P, 2A=3P, A=2P and 2A=Pp where p is a prime number. The Diophantine equations related with the area and circumference of the circle have been discussed in two different cases. The Diophantine equations related with the area and circumference of the cardioid has been discussed in three cases, namely, $A=\pi P$, $A=3\pi P$, and $16A=3\pi P$. The Diophantine equations related with the surface area and volume of the sphere have been discussed in three cases, namely, V=S, 3V=S and V=nS. The Diophantine equations related with the surface area and volume of the cylinder have been discussed in two different cases, i.e. V=C and V=S where V, C and S are respectively the volume, the curved surface and the total surface of the cylinder. The Diophantine equations related with the surface area and volume of the cone have been discussed in one case only when V=S. The Diophantine equations related with the surface area and volume of the rectangular parallelepiped have been discussed in four cases. The Diophantine equations related with the surface area and volume of regular polyhedrons have also been discussed. The Diophantine equations related with the surface area and volume of cuboid have been discussed in two cases. The Diophantine equations related with the surface area and volume of tetrahedron has been discussed in three cases, The Diophantine equations related with the surface area and volume of octahedron have been discussed in

three cases. The Diophantine equations related with the surface area and volume of dodecahedron an icosahedron have also been discussed.

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